Imperial College London

Lecture 7

Systems & Laplace Transform

Peter Cheung
Dyson School of Design Engineering

URL: www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/
E-mail: p.cheung@imperial.ac.uk

In this lecture, I will introduce **the idea of a system** to which we apply signals at the input and produce signals at the output. Any physical setup can take on a "system" view. Engineers model the system using mathematics. The main goal of system analysis is to be able predict its behaviour under different conditions. In so doing, we can design modifications to the system to give us desirable behaviour.

One of the most useful mathematical tools to analyse and thus, predict, systems is the **Laplace transform**. This lecture will introduce the theory of Laplace transform and show how it may be used to model systems as **transfer functions**.

10 things you have learned about signals (1)

1. Signals can be represented in **time domain** or **frequency domain**.
2. Any signal can be made up from **weighted sum of sinusoidal** signals.
3. A sinusoid at frequency ω and amplitude A can be an everlasting sine wave ($A \sin \omega t$), cosine wave ($A\cos \omega t$) or exponential ($A/2\ e^{j\omega t}$). Furthermore, two sinusoids at different frequencies have **NOTHING in common**.
4. For a **time-limited** signal, moving between time and frequency domain is done through **Fourier Transform**.
5. A **periodic signal** is represented in the frequency domain in **Fourier series**, where the fundamental frequency f_0 is 1/period of the signal, and all the other frequency are integer multiple of f_0.

Up to now, we have been focusing on the processing of electrical signals. In five short lectures, we have covered quite a lot of ground. It is therefore time to review what you have learned so far. Here are the TEN key teachings of what we have covered up to now:

1. **Signals in time-domain and frequency-domain views** – This is fundamental to signal processing. Depending on what you want to do with the signal, processing in one of the two domains will proof beneficial. A good example is shown earlier when a sinewave is corrupted by noise. In time-domain, it looks a mess. In frequency-domain, the energy is spread over the entire spectrum and therefore the sinewave is not "masked" by the noise.
2. **Any signal can be represented by weighted sum of sinusoids** – This is the essence of Fourier transform, and it is how we convert from time domain to frequency domain.
3. **Sinusoid as sine, cosine or exponential functions** – Sinusoids form the "building blocks" of signals in frequency domain. If you project a sinewave of one frequency onto another sinewave of a different frequency, no matter how close they are in frequency, the projection is zero. This implies that the two sinewaves are "orthogonal" and they have nothing in common. This is also why sinusoids form good building blocks.
4. Fourier Transform – converts a time-limited signal with finite energy from time-domain to frequency-domain.

$$X(\omega) = \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

10 things you have learned about signals (2)

6. You must sample a signal at a sampling frequency f_s which is **at least twice** that of the maximum signal frequency f_{max}: $f_s \geq 2*f_{max}$.
7. When **sampling signal** at f_s, the **spectrum** of the original signal is **repeated** at EVERY multiple of sampling frequency, i.e $\pm\ nf_s$, n = 1, 2, 3...
8. If you sample a signal which has a frequency component higher than fs/2, **aliasing** occurs (which results in **spectral folding**).
9. When you **extract** a portion of a signal, you effectively multiply the signal with a **rectangular window**, which results in spreading of energy to neigbouring frequency components. This is known as "**leakage**".
10. You can **reduce** this **leakage** by multiplying your signal with a **special window** function which has smooth instead of shape edges.

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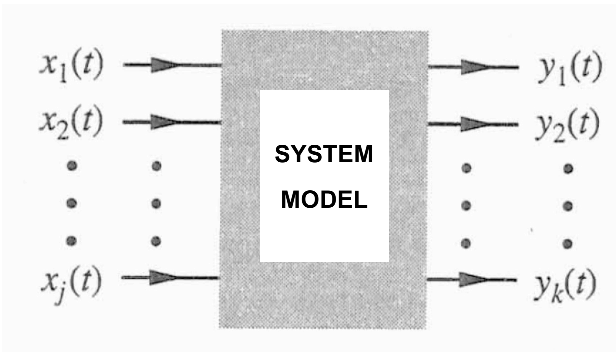
5. **Periodic signal uses Fourier series in frequency domain** – The fundamental frequency $f_0 = 1/T_0$, where T_0 is the period of the signal, and all other components are called **harmonics**, and they are at integral multiples of f_0.
6. **Sampling theorem** – One must sample at f_s samples per second, which is **at least TWICE** that of the maximum frequency of the signal fmax: $f_s \geq 2*fmax$.
7. **Spectrum of a sample signal** – When you sample a signal, the spectrum of the continuous time signal get repeated indefinitely at multiple of f_s, i.e. at $\pm n\ f_s$, where n is all integers except 0: ± 1, ± 2 ....
8. **Sampling a signal too slowly corrupts it through aliasing** – If you use a sampling frequency fs which is lower than 2*fmax, aliasing, i.e. spectral folding occurs and this will corrupt the signal in a way that you cannot go back to continuous time without error.
9. **Rectangular windows** – When extracting a portion of a signal to analyse, you are effectively multiplying the signal with a **rectangular window**. This results in spectral spreading and leakages – signal energy leaked to its neighbouring frequency components.
10. **Better to use window functions with smooth edges** – Leakages can be reduced significantly by using other they of windowing functions, such as Hamming and Hanning windows.

What are Systems?

- Systems are used to **process signals** to **modify** or **extract information**
- Physical systems – characterized by their **input-output relationships**
- E.g. electrical systems are characterized by voltage-current relationships for components and the **laws of interconnections** (i.e. Kirchhoff's laws)
- From this, we derive a **mathematical model** of the system
- "**Black box**" model of a system:



L1.6

Here is a general view of a SYSTEM. It processes signals from the input $x_j(t)$ and produces signals $y_k(t)$ at the output.

What we are attempting to do in this module is to learn how to **characterize** and **model** the **input-to-output relationship**. For example, we have already learned to calculate the relationship between output voltage and input voltage in an operational amplifier from your Year 1 Electronics 1 module.

Generally, we use mathematics to model the system behaviour, and produce some form of equations relating $y_k(t)$ to $x_j(t)$.

Since we don't really care what is exactly inside the system beyond this input-output relationship, we call this a "**Black box**" model of the system.

Linear Systems (1)

- A **linear system** exhibits the **additivity** property:

if $x_1 \longrightarrow y_1 \quad x_2 \longrightarrow y_2$ then $x_1 + x_2 \longrightarrow y_1 + y_2$

- It also must satisfy the **homogeneity** or **scaling** property:

if $x \longrightarrow y$ then $kx \longrightarrow ky$

- These can be combined into the property of **superposition**:

if $x_1 \longrightarrow y_1 \quad x_2 \longrightarrow y_2$ then $k_1x_1 + k_2x_2 \longrightarrow k_1y_1 + k_2y_2$

- A non-linear system is one that is NOT linear (i.e. does not obey the principle of superposition)

L1.7-1

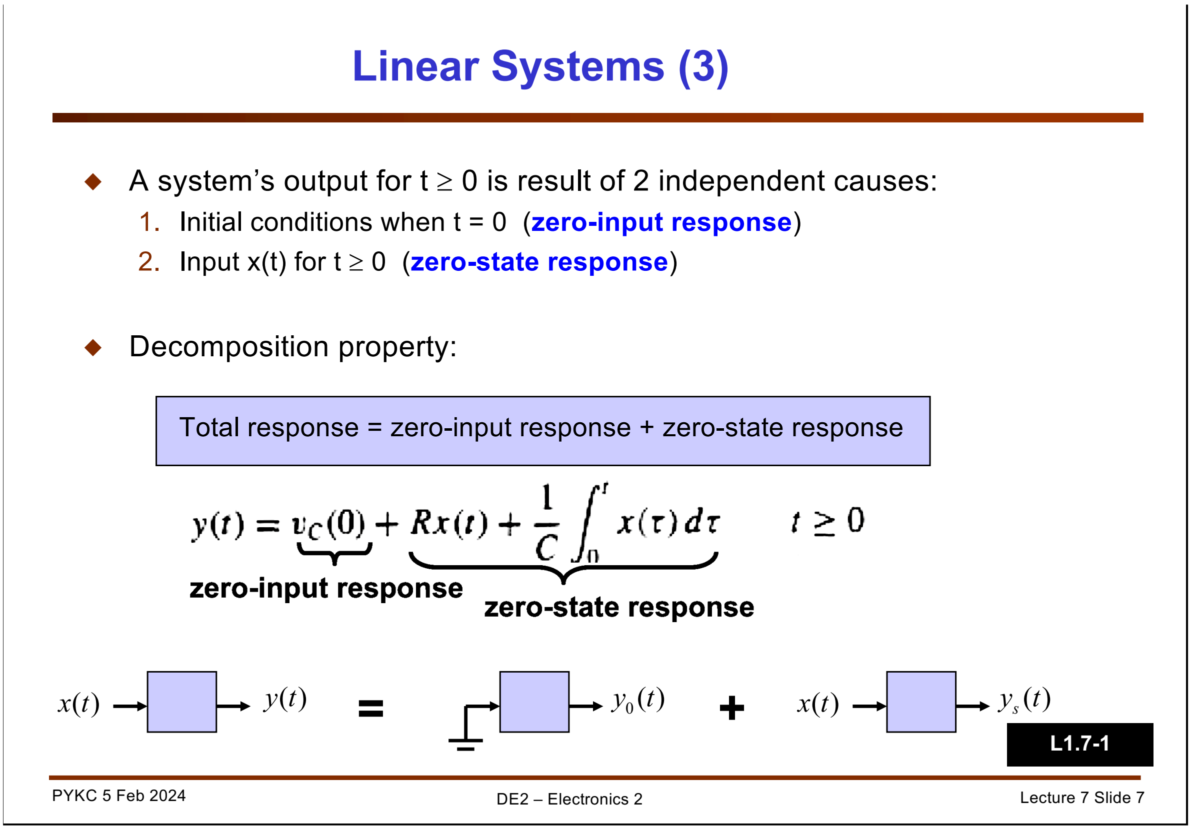
One of the most important property of any system is **linearity**. A linear system exhibits two important properties: 1) **additive**: if x1 leads to y1, x2 leads to y2, then x1+x2 leads to y1+y2; 2) **scaling**: if x leads to y, kx leads to ky.

These two properties can be combined to form the general form of **superposition**, a principle that we have already covered extensively last year.

Many physical systems are NOT inherently linear. For example, we have already considered that our ears are sensitive to sound volume in a logarithmic manner. An incandescent light bulb produce light output as a quadratic function (i.e. square) of the input voltage.

However, we can usually approximate a non-linear system as linear over a range of signal, particularly if the range is small. Therefore, we often perform the so-called "**small signal analysis**", restricting the signal to perturbation around a certain operating point.

We will examine this in Lab 3 in more details later.

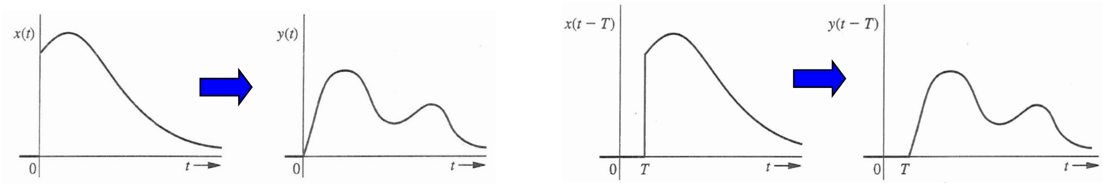


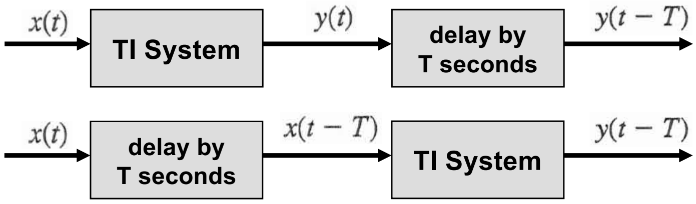
Now it is important to appreciate that given a system, the output response is made up of two parts:

1. The initial condition, which is also called the zero-input response. This is the system behaviour before any input is applied (as if the input is grounded).
2. The zero-state response. This is the the system behaviour of the system to the input assuming that the internal state (such as the capacitor voltage) are all initially zero.

Time-Invariant Systems

- **Time-invariant system** is one whose parameters do not change with time:





- Linear time-invariant (**LTI**) systems – main type of systems for this course.

L1.7-2

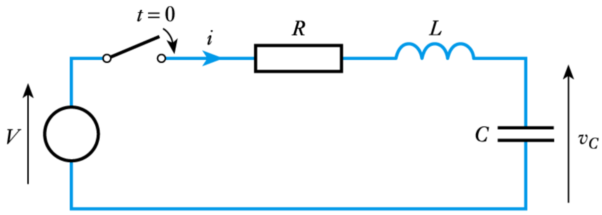
Another important classification of any systems is time-invariant vs time-variant.

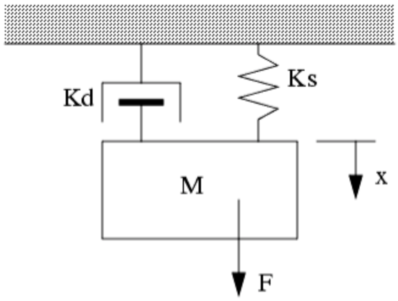
A time-invariant system means that the characteristic is NOT change (invariant) over time. It is fixed and no dependent on when you use the system, today, tomorrow or next year.

In this module, we only consider systems that are LINEAR, and TIME-INVARIANT, and call this LTI system for short.

System modelling using ODEs

- Many systems in electrical and mechanical engineering where input and output are related by **ordinary differential equations (ODEs)**
- For example:





$$v_L(t) + v_R(t) + v_c(t) = V$$

$$LC\frac{d^2 v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C = V$$

$$M\ \ddot{x}(t) + K_d\dot{x}(t) + K_s x(t) = F(t)$$

L1.8

You are familiar with modeling systems with differential equations. Assuming that all voltages and currents were 0 for t < 0. At t = 0, the switch closes. We are interested in find out v_c(t) as a function of time.

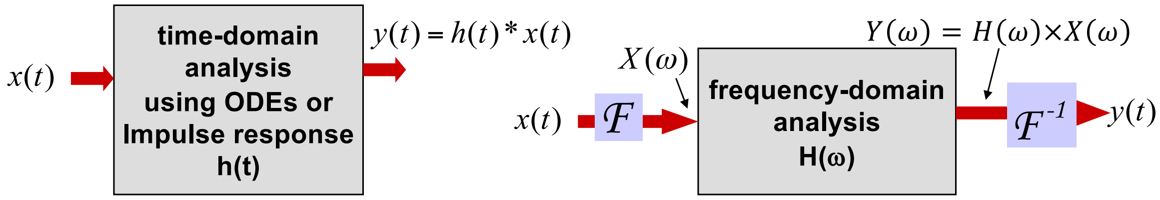
You can easily write an equation as shown by summing the voltage around the loop (Kirkoff's voltage law – voltage around a loop in a circuit sums to zero). This provides us with a differential equation, which can be solved for v_c(t).

Similar, consider a mechanical system with a mass M, hanging from the ceiling with a damper with damping coefficient Kd and a spring with a Young's coefficient Ks. If you apply a force F(t) the mass, what is x(t)?

Summing all the forces together in the vertical direction, we get the differential equation shown. The gravitation force is proportional to d^2x/dt^2. The force of the damper if proportional to dx/dt. The force on the spring is proportional to x(t) itself.

These differential equations capture the behaviour of the systems from which we can predict the output for any input, whether it is under rapid change (**transient** behaviour) or the input is fixed (**steady-state** behaviour). Although modeling systems as differential equation works, solving ODE is a bit tedious. Laplace transform is a method to solve ODEs without pain!

System Analysis in time and frequency domains



- Analyse system using differential equations or using the system's **impulse response** h(t) (later lecture)
- Analyse system using **frequency response** H(ω)
- Analyse system behaviour in time-domain via solving differential equations can be tedious.
- Could use impulse response and **convolution** (later topic), but could be expensive.
- Using Fourier transforms and frequency response to analyse (and predict behaviour of) a system has limitations.
- Frequency response is only useful in predicting **steady-state behaviour** of a system, not transient behaviour.
- Alternative – use Laplace transform to transform both system and signals to the complex Laplace variable, the s-domain.

Before we consider Laplace transform theory, let us put everything in the context of signals being applied to systems.

If we take a time-domain view of signals and systems, we have the top left diagram. The input x(t) is a function of time (i.e. a waveform you see on a scope), and the system is modeled as ODEs. Alternatively you may also model the time-domain system through its response to an impulse at the input. The system response to an impulse is known as "**impulse response**" and is usually represented as h(t). We will be covering **impulse response** in a later lecture.

In time-domain analysis, you get y(t) either by solving the ODEs or you could derive y(t) from x(t) and h(t) through an operation known as "**convolution**". This is again something that will be covered later in this module.

However, if you operate in the frequency domain (from now on, I will drop the hyphen), we take the Fourier transform of the input signal: x(t) ➔ X(ω). We then model the system with its frequency response H(ω). The output (in the frequency domain) Y(ω) is given by Y(ω) = X(ω) x H(ω), a simple multiplication.

In other words, the frequency response H(ω) is a model of how the system passes (or suppresses) different frequency components in the signal X(ω). This is the process whereby you adjust your mobile phone playing music to emphasize low frequencies (bass) to get stronger beats in pop music, or to emphasize higher frequencies (treble) to gain more clarity in classical music.

Laplace Transform (1)

- Laplace Transform is a method that converts differential equations in time-domain into algebraic equations in complex Laplace variable s-domain.
- Definition of Laplace Transform $\mathcal{L}$ is:

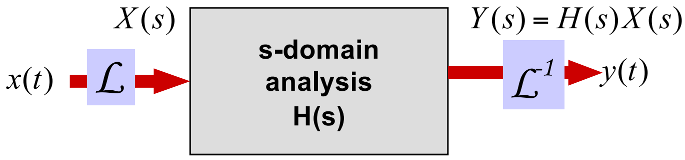
$$\mathcal{L}[X(t) = X(s) = \int_0^{\infty} x(t)e^{-st}dt$$

Fourier Transform

$$\mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$s = \alpha + j\omega$$

- Once transformed to the s-domain, analysis and prediction of the system becomes easy if we know the system's characteristic H(s), which is also called the **transfer function** (more later)



L4.1

Laplace transform is in someway similar to Fourier Transform. However, it is more general, and arguably more powerful.

It converts differential equations in the time domain into algebraic equations in another domain with a complex Laplace variable s. Let us call this the s-domain.

The mathematical definition of the general Laplace Transform (also called bilateral Laplace Transform) is:

$$\mathcal{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\, dt$$

where $s = \alpha + j\omega$

For this course, we assume that the signal and the system are both causal, i.e. x(t) = 0 for all t < 0. Therefore we get the equation shown in the slide, where the limits of integration is from 0 and NOT -∞.

Similar to Fourier domains, we can transform input signal x(t) to the Laplace or s-domain as X(s), and we can model the system in the s-domain using its response H(s). This is also called the Transfer Function. If you known X(s) and H(s), then the output in the s-domain Y(s) = H(s) X(s) – very similar to the Fourier analysis we did before.

We will consider the relationship (similarity) between Fourier transform and Laplace transform later. For now, you can regard Fourier transform is a special case of Laplace transform. So Laplace is more general. Laplace transform becomes Fourier transform if $s = \alpha + j\omega$ where α=0. Then $s = j\omega$.

Laplace Transform (2)

- Laplace Transform obeys laws of **linearity**:

$$\mathcal{L}\left[\beta_1 x_1(t) + \beta_2 x_2(t)\right] = \beta_1 \mathcal{L}[x_1(t)] + \beta_2 \mathcal{L}[x_2(t)]$$

- The Laplace transform of **an impulse function**:

$$\mathcal{L}[\delta(t)] = \int_0^\infty \delta(t) e^{-st} dt = 1 \qquad \text{for all } s$$

$$\mathcal{L}[\delta(t)] \quad \Leftrightarrow \quad 1$$

- The Laplace transform of a **unit step function**:

$$\mathcal{L}[u(t)] = \int_0^\infty u(t) e^{-st} dt = \int_0^\infty e^{-st} dt$$

$$= -\frac{1}{s} e^{-st}\Big|_0^\infty = \frac{1}{s} \qquad \text{Re } s > 0$$

$$\mathcal{L}[u(t)] \quad \Leftrightarrow \quad \frac{1}{s}$$

L4.1

Before we go any further, let us consider the Laplace transforms of interesting signals and functions.

First, you must remember that Laplace transform, just like Fourier, obeys the law of linearity – it is a linear tranform.

Now let us consider the Laplace transform of an impulse δ(t). This simple integration shows that:

$$\mathcal{L}[\delta(t)] \quad \Leftrightarrow \quad 1$$

This is similar to the case of Fourier transform shown in Lecture 4, slide 7.

The Laplace transform of a unit step signal u(t) is $\frac{1}{s}$. Again you can derive this through simple integration. Remember that e^{-st} ➔ 0 when t ➔∞.

Laplace Transform (3)

- Laplace Transform of $e^{at}u(t)$:

$$\mathcal{L}[e^{at}u(t)] = \int_0^\infty e^{at}e^{-st}dt$$
$$= \int_0^\infty e^{-(s-a)t}dt = \frac{1}{s-a}$$

$$\mathcal{L}[e^{at}u(t)] \Leftrightarrow \frac{1}{s-a}$$

- Laplace Transform of $\cos \omega_0 t\, u(t)$:

$$\mathcal{L}[\cos\omega_0 t\, u(t)] = \frac{1}{2}\mathcal{L}[e^{j\omega_0 t}u(t) + e^{-j\omega_0 t}u(t)]$$
$$= \frac{1}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right] = \frac{s}{s^2+\omega_0^2}$$

$$\mathcal{L}[\cos\omega_0 t u(t)] \Leftrightarrow \frac{s}{s^2+\omega_0^2}$$

L4.1

Now consider Laplace transform of a causal exponential signal e^{at} u(t). (Note that multiplying e^{at} by u(t) makes the signal causal because u(t) chops off everything where t < 0.)

Again simple integration yields the result you see here.

From this, we can also derive the Laplace transform for a causal cosine signal at frequency ω_0.

Laplace Transform (4)

- Laplace Transform of a **differentiator** $\dot{x}(t) = \frac{dx(t)}{dt}$:

$$\mathcal{L}[\frac{dx(t)}{dt}] = \int_{t=0}^{\infty} \frac{dx(t)}{dt} e^{-st}\, dt$$

- It can be shown (using integration by parts) that this result in:

$$\mathcal{L}[\dot{x}(t)] = sX(s) - x(0)$$

- If x(0) = 0 (i.e. zero initial condition), then $\mathcal{L}[\dot{x}(t)] = sX(s)$

- Therefore, differentiation in the time domain is multiplication by s in the s-domain:

$$\frac{d}{dt} \overset{\mathcal{L}}{\longleftrightarrow} s$$

We can also derive the Laplace transform for a function. For example, what is the LT of a differentiation function d/dt?

As shown here, the result is also pretty simple. x(0) is the initial value of x at t = 0. If x(0) = 0, i.e. zero initial condition, then L(dx(t)/dt) = s X(s). This is a very important result.

Laplace Transform (5)

- Laplace Transform of an **integrator** $\int_{\tau=0}^{t} x(\tau)d\tau$:

$$\text{Let } g(t) = \int_{\tau=0}^{t} x(\tau)d\tau$$
$$\text{then } x(t) = \frac{dg(t)}{dt}, \quad \text{and} \quad g(0) = 0$$

- From last slide

$$\mathcal{L}[x(t)] = \mathcal{L}[\dot{g}(t)] = sG(s) - g(0) = sG(s)$$

- Therefore

$$\mathcal{L}[g(t)] = \frac{1}{s}X(s)$$

- Therefore, integration in the time domain is multiplication by 1/s in the s-domain:

$$\int_{t=0}^{t} \overset{\mathcal{L}}{\longleftrightarrow} s^{-1}$$

Similarly, we can compute the Laplace transform of the integration function. This is slightly more complicated.

We first express the integration of x(t) as g(t): $g(t) = \int_{\tau=0}^{t} x(\tau)d\tau$

This leads to: $x(t) = \frac{dg(t)}{dt}, \quad \text{and} \quad g(0) = 0$

If we now take Laplace transform on both sides, we get:

$$\mathcal{L}[x(t)] = \mathcal{L}[\dot{g}(t)] = sG(s) - g(0) = sG(s)$$

Therefore, LT of an integrator is the same as multiplying the input X(s) by 1/s in the s-domain.

Laplace transform Pairs (1)

- Finding inverse Laplace transform requires integration in the complex plane – beyond scope of this course.
- So, use a Laplace transform table (analogous to the Fourier Transform table).

No.	$x(t)$	$X(s)$
* 1	$\delta(t)$	1
* 2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$

L4.1

The table of Laplace transform pairs (going both directions) is taken from Lathi's book. The first TWO shown here are useful, particularly for signals and systems.

The first pair is the impulse function. The LT is the constant 1.

Pair 2 is the LT of the unity step function, and we have seen in L6 S13 that this is computed to be 1/s.

Laplace transform Pairs (2)

	No.	$x(t)$	$X(s)$
*	5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$
	6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$
	7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
*	8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
*	8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
*	9a	$e^{-at}\cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
*	9b	$e^{-at}\sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$

L4.1

Pair 5 here is MOST important. You will find that most systems will have terms in the form of $\frac{1}{s-\lambda}$ in the s-domain. The time domain equivalent of this is a causal exponential function $e^{\lambda t}u(t)$. The unity step function u(t) makes this causal, meaning that it is zero for t < 0. The term $e^{\lambda t}$ is the general solution for most differential equations. It represents the natural response of many physical systems.

Pairs 8a and 8b are also important because they represent the LT of causal sine and cosine waveforms.

Finally, 9a and 9b represents exponential decaying, causal sine and cosine, something that occurs frequently in the physical world.

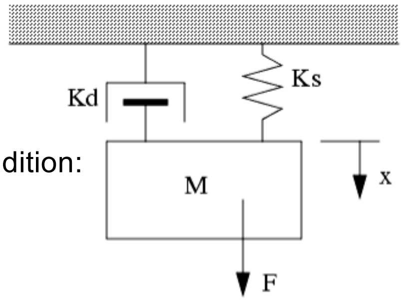
Laplace Transform vs Differential Equations

- Since $\mathcal{L}\left[\frac{x(t)}{dt}\right] = sX(s)$

  we can generalise higher order differential as: $\frac{d^k}{dt^k} \overset{\mathcal{L}}{\leftrightarrow} s^k$

- Therefore, consider the mechanical system in slide 10:

$$M\ \ddot{x}(t) + K_d\dot{x}(t) + K_s x(t) = F(t)$$



- Apply Laplace transform assuming zero initial condition:

$$Ms^2X(s) + K_d sX(s) + K_s X(s) = F(s)$$

$$(Ms^2 + K_d s + K_s)X(s) = F(s)$$

$$\Longrightarrow H(s) = \frac{X(s)}{F(s)} = \frac{1}{(Ms^2 + K_d s + K_s)}$$

H(s) is TRANSFER FUNCTION

Now we are ready to generalize. Assuming zero initial condition, L[dx/dt] = sX(s), it follows that L[d^2x/dt^2] is $s^2X(s)$ ….. L[d^kx/dt^k] is $s^k X(s)$.

So let us take our mechanical system previously considered in Slide 10. The second-order differential equation:

$$M\ \ddot{x}(t) + K_d\dot{x}(t) + K_s x(t) = F(t)$$

Can be converted to the Laplace s-domain (zero initial condition) as:

$$Ms^2X(s) + K_d sX(s) + K_s X(s) = F(s)$$

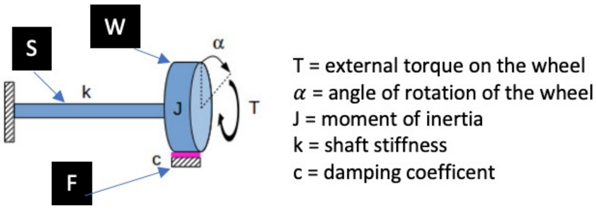
Re-arrange this a bit, and express this as OUTPUT/INPUT in the s-domain, we get:

$$H(s) = \frac{X(s)}{F(x)} = \frac{1}{Ms^2 + K_d s + K_s}$$

This is a very important results. H(s) is known as Transfer function, and it characterizes the system in the s-domain as a 2nd order polynomial function in the complex Laplace variable s. This is an algebraic equation. Since Y(s) = H(s) X(s), a simple multiplication, we can predict the output by simple algebraic calculations. No more fiddling with differential equations!

Using Laplace Transform to model a system

- Here is another mechanical system with a wheel (taken from last year's examination paper):



- The relationship between the wheel angle α and the external torque T is given by the following equation:

$$T - k\alpha - c\frac{d\alpha}{dt} - J\frac{d^2\alpha}{dt^2} = 0$$

- Apply Laplace transform assuming zero initial condition:

$$T(s) - k\alpha(s) - cs\alpha(s) - Js^2\alpha(s) = 0$$

Hence,

$$H(s) = \frac{\alpha(s)}{T(s)} = \frac{1}{Js^2 + cs + k}$$

A torsion system with a heavy wheel W has a moment of inertia J. It is connected to a stationary anchor through a shaft S with a shaft stiffness of k as shown in Figure Q4. The movement of the wheel is damped by a friction pad F with a damping coefficient of c. An external torque T is acting on the wheel in the direction shown. The angle of rotation of the wheel α is measured from its stationary condition. The relationship between the wheel angle α and the external torque T is given by the following equation:

$$T - k\alpha - c\frac{d\alpha}{dt} - J\frac{d^2\alpha}{dt^2} = 0$$

Instead of using differential equation to model the system, we can take Laplace Transform on both sides of the equation:

$$T(s) - k\alpha(s) - cs\alpha(s) - Js^2\alpha(s) = 0$$

Now we can derive the transfer function. $H(s) = \frac{ouput(s)}{input(s)} = \frac{\alpha(s)}{T(s)}$

Hence, we turn a differential equation in time domain to an algebraic equation in complex frequency s-domain.

$$H(s) = \frac{\alpha(s)}{T(s)} = \frac{1}{Js^2 + cs + k} = \frac{1}{k}\left[\frac{\frac{k}{J}}{s^2 + \frac{c}{J}s + \frac{k}{J}}\right]$$

Three Big Ideas

1. Laplace transform is useful for analysing systems. It maps time domain behaviour to the complex frequency s-domain where $s = \alpha + j\omega$. This contrasts with Fourier transform which maps to frequency (or ω) domain.

2. Laplace transform converts mathematical models of real systems described using differential equations in time domain to algebraic equation in s-domain. This is possible because:

$$\mathcal{L}\left(\frac{d}{dt}\right) = s \quad \text{and} \quad \mathcal{L}\left(\frac{d^2}{dt^2}\right) = s^2$$

3. Transfer function of a system H(s) is the Laplace transform of the output signal Y(s) divided by the Laplace transform of the input signal X(s):

$$H(s) = \frac{Output\ Y(s)}{Input\ X(s)}$$

Here are the three things that you should know and remember, and even better, understand.